CHAPTER 5 APPLICATIONS OF INTEGRATION

5.1 Geometrical Interpretation-Definite Integral
(page 362)

5.2 Area of a Region (page 369)
5.2.1 Area of a Region Under a Graph (page 369)

Figure 5.7 shows the region bounded by the curve \( y = f(x) \), the \( x \)-axis, and the lines \( x = a \) and \( x = b \). This region is located above the \( x \)-axis. The area of this region is given by

\[
\int_a^b f(x) \, dx
\]
Figure 5.8 shows the region bounded by the curve $y = g(x)$, the $x$-axis, and the lines $x = a$ and $x = b$. This region is located below the $x$-axis. The definite integral $\int_a^b f(x) \, dx$ has a negative value. Since the area is always a positive quantity, the area of this region is written as

$$\left| \int_a^b g(x) \, dx \right|$$

Figure 5.9 shows the region bounded by the curve $x = u(y)$, the $y$-axis, and the lines $y = c$ and $y = d$. This region is located on the right hand side of the $y$-axis. The area of this region is given by

$$\int_c^d u(y) \, dy$$

**Example 5.3 (page 370):**

Find the area of the region bounded by the curve $y = 2 - x^2$, the $x$-axis and the lines $x = 0$ and $x = 1$.

**Example 5.4 (page 371):**
Find the area bounded by the lines \( y = 2 - x \), \( x = 3 \), \( x = 4 \) and \( x \)-axis.

**Example 5.8 (page 374):**

Find the area of the region in the first quadrant bounded by the curve \( y = \frac{2}{x} \), \( y \)-axis with lines \( y = 2 \) and \( y = 4 \).

### 5.2.2 Area of the Region between Two Curves

**Definition 5.2 (Area Between Two Curves)**

If \( f(x) \) and \( g(x) \) are continuous in the interval \([a, b]\) and \( g(x) \leq f(x) \) for all \( x \) in \([a, b]\), then the area of the region
bounded by the curves \( y = f(x) \) and \( y = g(x) \) between the lines \( x = a \) and \( x = b \) is given by
\[
A = \int_{a}^{b} \{ f(x) - g(x) \} \, dx.
\]

**Example 5.10 (page 376):**
Find the area of the region bounded by the curve \( y = x^2 + 2 \) and the lines \( y = -x \), \( x = 0 \) and \( x = 1 \).

**Example 5.15 (page 380):**
Find the area of the region bounded by the curves \( y^2 = 3 - x \) and the line \( y = x - 1 \).

**Definition 5.3 (Area Between Two Curves about y-Axis) (page 383)**
If \( u(y) \) and \( v(y) \) are continuous in the interval \([c, d]\) and \( v(y) \leq u(y) \) for all \( y \) in \([c, d]\), then the area of the region bounded by the curves \( x = u(y) \) and \( x = v(y) \) between the lines \( y = c \) and \( y = d \) is given by
\[
A = \int_{c}^{d} \{ u(y) - v(y) \} \, dy.
\]

**Example 5.17 (page 383):**
Find the area of the region bounded by the curves \( y^2 = 3 - x \)
and a line \( y = x - 1 \).

### 5.3 Volume of Revolution (page 387)

If a plane region is revolves about a line then a solid object
is generated which is called the **solid of revolution**, and the
line is called the **axis of revolution**.

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**Definition 5.4 (Volume of Revolution about x-Axis)
(page 388)**

Let \( f(x) \) be a non-negative and continuous function in the
interval \([a, b]\). If the region between this curve, the x-axis
and the lines \( x = a \) and \( x = b \) revolves \(360^\circ\) about the x-axis,
then the volume of the solid generated is

\[
V = \pi \int_a^b [f(x)]^2 \, dx.
\]

**Example 5.20 (page 388):**
Find the volume of the solid of revolution when the region bounded by the parabola \( y = 2\sqrt{x} \) and \( x \)-axis within the interval \([0, 4]\) revolves \(360^\circ\) about the \(x\)-axis.

**Definition 5.5 (Volume of Revolution about \(y\)-Axis)**  
* (page 390)  
Let \( u(y) \) be a non-negative and continuous function in the interval \([c, d]\). If the region bounded by \( x = u(y) \), the \(y\)-axis and the lines \( y = c \) and \( y = d \) revolves \(360^\circ\) about the \(y\)-axis, the volume of the solid generated is  
\[
V = \pi \int_{c}^{d} [u(y)]^2 \, dy.
\]

**Example 5.23 (page 390):**  
Find the volume of the solid of revolution when the region bounded by the curve \( y = x^2 + 1 \), the lines \( y = 1, \ y = 2 \) and the \(y\)-axis revolves \(360^\circ\) about the \(y\)-axis.

**Definition 5.6 (Volume of Revolution about \(x\)-Axis between two Curves)**  
* (page 391)  
Let \( f(x) \) and \( g(x) \) be non-negative and continuous functions in the interval \([a, b]\) and \( g(x) \leq f(x) \) for all \(x\) in the interval \([a,
b]. The volume of revolution when the region bounded by 
\( y = f(x), \ g(x), \ x = a \) and \( x = b \), revolves 360° about the x-axis is
\[
V = \pi \int_a^b \left( [f(x)]^2 - [g(x)]^2 \right) dx.
\]

**Example 5.24 (page 391):**
Find the volume of the solid of revolution when the region bounded by the curve \( y^2 = 8x \) and \( y = x^2 \) revolves at 360° about the x-axis.

**Definition 5.7 (Volume of Revolution about y-Axis Between Two Curves) (page 392)**
Let \( u(y) \) and \( v(y) \) be a non-negative and continuous function in the interval \([c, d]\) and \( v(y) \leq u(y) \) for all \( y \) in the interval \([c, d]\). The volume of the solid generated when the region bounded by the curves \( x = u(y), \ x = v(y), \ y = c \) and \( y = d \) revolves 360° about the y-axis is
\[
V = \pi \int_c^d \left( [u(y)]^2 - [v(y)]^2 \right) dy.
\]

**Example 5.25 (page 392):**
Find the volume of the solid of revolution when the region bounded by the curve \( y^2 = 8x \) and \( y = x^2 \) revolves at 360° about the x-axis.
Exercise at home: (Tutorial 10)

(Page 385) Quiz 5B: no. 1, 3, 4, 7.

(Page 394) Quiz 5C: no. 1(a), 1(c), 2(b), 2(e).

(Page 395) Exercise 5: no. 23, 33, 43.